# DESUBLIMATION OF PURE WATER VAPOR AT A FLAT SURFACE

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Results are shown of a theoretical and experimental study concerning the desublimation of water vapor at a flat surface.

Transformation of water vapor into the solid state (desublimation) occurs always in the presence of some noncondensing gases. In the most important practical cases, however, the quantity of such inclusions near the phase-transition interface is so small that they do not appreciably affect the process. The authors consider here just such a case, under the assumption that the pressure of the ambient vapor and the temperature of the heat sink surface are maintained constant throughout the process.

A peculiarity of the desublimation process, in contrast to vapor condensation into the liquid state, is its transiency caused by displacements of the phase-transition boundary. For an analytical study of the process, we will first determine the temperature characteristics of the interphase surface. Disregarding the specific heat of ice and vapor, and assuming a condensation factor in the Hertz–Knudsen equation of unity, we obtain the following expression for the temperature at that surface:

$$\lambda_1 \frac{T_{ph} - T_0}{\xi} = 1.185 \cdot 10^{-2} \sqrt{\frac{\mu}{2\pi T_{ph}}} (p - p_{ph})\, r. \tag{1}$$

The solution to this equation is presented graphically in Fig. 1. Evidently, the difference between saturation temperature and phase-transition interface temperature is very small (not more than 5% of the total temperature drop across the ice layer) and decreases fast with a thicker ice layer. We may therefore assume $T_{ph} = T_s$. With this assumption, the problem can be formulated mathematically as follows:

$$\frac{\partial T_1}{\partial \tau} = a_1 \frac{\partial^2 T_1}{\partial x^2} \quad (\tau > 0;\ 0 < x < \xi), \tag{2}$$

$$\frac{\partial T_2}{\partial \tau} - \left(\frac{\rho_1}{\rho_2} - 1\right) \frac{\partial T_2}{\partial x} \cdot \frac{d\xi}{d\tau} = a_2 \frac{\partial^2 T_2}{\partial x^2} (\tau > 0;\ \xi < x < \infty). \tag{3}$$

The boundary conditions are

$$T_1 = T_2 = T_s = \text{const} \quad \text{at} \quad x = \xi(\tau), \tag{4}$$

$$T_1 = T_0 = \text{const} \quad \text{at} \quad x = 0 \ \text{and} \ \tau \geqslant 0, \tag{5}$$

$$T_2 = T_v = \text{const} \quad \text{at} \quad x \to \infty, \tag{6}$$

$$\lambda_1 \frac{\partial T_1}{\partial x} - \lambda_2 \frac{\partial T_2}{\partial x} = \rho_1 r \frac{d\xi}{d\tau} \quad \text{at} \quad x = \xi, \tag{7}$$

with known solutions [1]:

$$T_1 = T_0 + \frac{T_s - T_0}{\operatorname{erf}\beta} \operatorname{erf} \frac{x}{2\sqrt{a_1 \tau}}, \tag{8}$$

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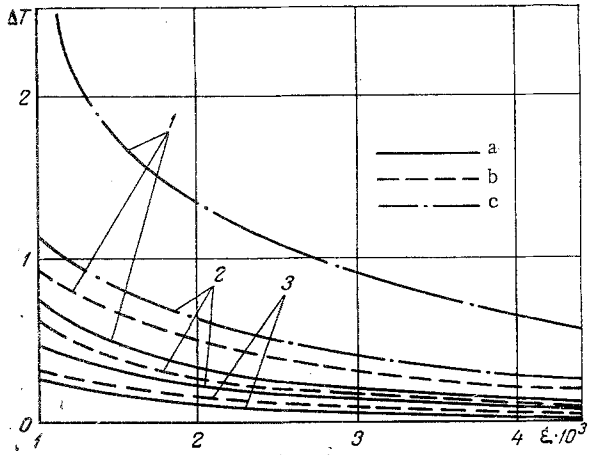


Fig. 1

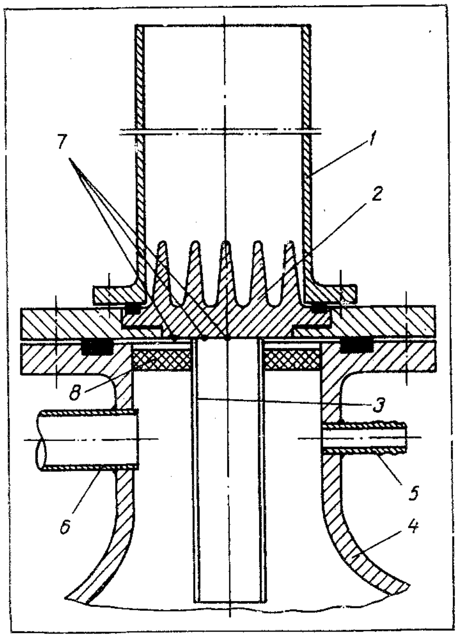


Fig. 2

Fig. 1. Temperature difference $\Delta T = T_S - T_\Phi$ as a function of the ice layer thickness $\xi$ (m), at various pressures p and temperatures $T_0$: 1) $T_0 = 213°K$; 2) 233; 3) 253; a) p = 4.5 mm Hg; b) 2.5; c) 0.5.

Fig. 2. Schematic diagram of the operating part of the test apparatus: 1) beaker; 2) heat sink element; 3) glass cylinder; 4) buffer space; 5, 6) connecting tubes; 7) thermocouple; 8) retaining ring.

$$T_2 = T_V - \frac{T_V - T_S}{\operatorname{erfc}\left(\beta \frac{\rho_1}{\rho_2}\sqrt{\frac{a_1}{a_2}}\right)} \operatorname{erfc}\left[\frac{x}{2\sqrt{a_2\tau}} + \left(\frac{\rho_1}{\rho_2} - 1\right)\beta\sqrt{\frac{a_1}{a_2}}\right]. \quad (9)$$

Factor $\beta$ is determined from the equation:

$$\frac{\exp(-\beta^2)}{\beta \operatorname{erf}\beta} - \frac{\lambda_2\sqrt{a_1}(T_V - T_S)\exp\left[-\beta^2\left(\frac{\rho_1}{\rho_2}\right)^2\frac{a_1}{a_2}\right]}{\beta\lambda_1\sqrt{a_2}(T_S - T_0)\operatorname{erfc}\left[\beta\frac{\rho_1}{\rho_2}\sqrt{\frac{a_1}{a_2}}\right]} \cdot \frac{r\sqrt{\pi}}{c_1(T_S - T_0)}. \quad (10)$$

The boundary displacement is determined from the relation

$$\xi = 2\beta\sqrt{a_1\tau}, \quad (11)$$

and the quantity of desublimated mass is found by the formula

$$m = 2\rho_1\beta\sqrt{a_1\tau}. \quad (12)$$

In order to verify these results, the authors undertook an experimental study. The test apparatus consisted of a vapor generator, a buffer space, a pressure measuring system, a main and an auxiliary condenser, and a model VN-2MG vacuum pump. A schematic diagram of the test segment is shown in Fig. 2. Vapor was generated from distilled water carefully degassed beforehand. The heat sink element was made of copper with an active surface area of $6.15 \cdot 10^{-4}$ m². In order to raise the heat-transfer rate, this element was finned on the coolant side. A mixture of solid carbon dioxide and alcohol served as the coolant.

The temperature of the desublimation surface was measured with three copper–constantan thermocouples. The total pressure in the buffer space was checked with a U-tube vacuometer filled with dibutyl phthalate.

The test was performed in the following sequence. First the system was evacuated down to $5 \cdot 10^{-3}$ mm Hg through the connecting tube 5. Then beaker 1 was filled with a mixture of alcohol and solid carbon

TABLE 1. Values of $\beta$ Calculated for Various Values of the Thermodynamic Parameters

| $T_0$ | $p$ | 4,5 | | | 3,5 | | | 2,5 | | | 1,5 | | | 0,5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $T_V$ | 273 | 293 | 333 | 273 | 293 | 333 | 273 | 293 | 333 | 273 | 293 | 333 | 273 | 293 | 333 |
| 213 | (10) | 0,151 | 0,150 | 0,149 | 0,144 | 0,143 | 0,140 | 0,138 | 0,137 | 0,136 | 0,122 | 0,121 | 0,120 | 0,104 | 0,104 | 0,102 |
| | (13) | 0,151 | — | — | 0,142 | — | — | 0,135 | — | — | 0,122 | — | — | 0,104 | — | — |
| | (14) | 0,152 | — | — | 0,144 | — | — | 0,139 | — | — | 0,123 | — | — | 0,105 | | |
| 223 | (10) | 0,137 | 0,136 | 0,135 | 0,130 | 0,129 | 0,128 | 0,125 | 0,124 | 0,123 | 0,109 | 0,109 | 0,108 | 0,088 | 0,088 | 0,086 |
| | (13) | 0,137 | — | — | 0,130 | — | — | 0,122 | — | — | 0,109 | — | — | 0,089 | — | — |
| | (14) | 0,138 | — | — | 0,131 | — | — | 0,126 | — | — | 0,109 | — | — | 0,089 | — | — |
| 233 | (10) | 0,123 | 0,122 | 0,121 | 0,116 | 0,115 | 0,114 | 0,110 | 0,110 | 0,109 | 0,093 | 0,092 | 0,091 | 0,068 | 0,068 | 0,067 |
| | (13) | 0,123 | — | — | 0,115 | — | — | 0,110 | — | — | 0,093 | — | — | 0,069 | — | — |
| | (14) | 0,124 | — | — | 0,116 | — | — | 0,111 | — | — | 0,093 | — | — | 0,069 | — | — |
| 243 | (10) | 0,106 | 0,105 | 0,104 | 0,098 | 0,097 | 0,0965 | 0,092 | 0,092 | 0,092 | 0,073 | 0,073 | 0,072 | 0,041 | 0,041 | 0,041 |
| | (13) | 0,106 | — | — | 0,098 | — | — | 0,092 | — | — | 0,073 | — | — | 0,042 | — | — |
| | (14) | 0,107 | — | — | 0,099 | — | — | 0,093 | — | — | 0,074 | — | — | 0,042 | — | — |

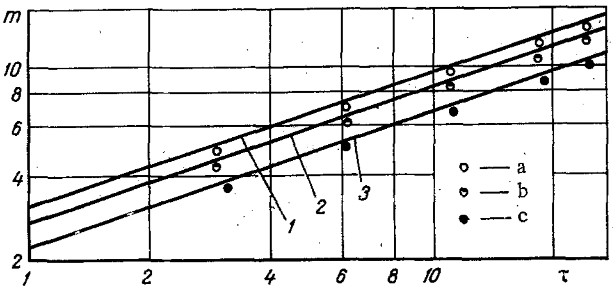


Fig. 3. Comparison between theoretical and experimental values of desublimated vapor mass (kg/m$^3$). Temperature of the heat sink surface 218°K. Calculated results: 1) p = 4.5 mm Hg; 2) 2.5; 3) 0.5. Test results: a) p = 4.5 mm Hg; b) 2.5; c) 0.5. Time $\tau \cdot 10^{-1}$ sec along the axis of abscissas.

dioxide. After a steady temperature had been reached (–55°C in our test), vapor was fed to the heat sink surface 2 and into the buffer space 4 through the connecting tube 6. The vapor supply was regulated so as to maintain a constant pressure in the buffer space throughout the process. After a definite interval of time, the system was shut off for measuring the thickness and the weight of the desublimate. The measurement error did not exceed 0.1 mm in thickness and 0.05 g in weight.

The test values (averages of five measurements) and the theoretical values of desublimated ice mass, per unit surface area, are shown in Fig. 3. The difference between tested and calculated values of desublimated ice does not exceed 10% here. In addition, we also studied the effect of vapor superheat on the process kinetics. Even at 100°C superheat was found not to affect the desublimation rate appreciably, in complete agreement with theory. Thus, our results indicate that the quantitative relations have been based on correct assumptions.

Considering that the calculation of $\beta$ from Eq. (10) presents certain difficulties, it is worthwhile to have simpler formulas for this purpose. The first simplified relation is easily derived by assuming $T_V = T_S$, when (10) becomes

$$\beta \exp \beta^2 \operatorname{erf} \beta = \frac{c_1 (T_s - T_0)}{r \sqrt{\pi}} . \tag{13}$$

Furthermore, by expanding both the exponential and the error function into series, with all except the first term omitted, we have

$$\beta = \sqrt{\frac{c_1 (T_s - T_0)}{2r}} . \tag{14}$$

This result can be obtained directly with a steady-state temperature field assumed in the ice layer.

In order to assess the feasibility of using formulas (13) and (14) instead of (10), factor $\beta$ was calculated by all three formulas with the values of the thermodynamic parameters corresponding to typical industrial sublimation apparatus. The results are shown in Table 1. It appears that $\beta$ can, within sufficient accuracy, be calculated by formula (14).

## NOTATION

$T_0$ is the temperature of the heat sink surface;
$T_{ph}$ is the temperature of the interphase boundary surface;
$T_s$ is the saturation temperature;
$T_v$ is the vapor temperature;
$\lambda$ is the thermal conductivity;
$\xi$ is the ice layer thickness;
r is the heat of desublimation;
$\mu$ is the molecular weight;
p is the ambient vapor pressure;
$p_{ph}$ is the vapor pressure on the interphase boundary surface;
$a$ is the thermal diffusivity;
$\rho$ is the density;
$\tau$ is the time;
x is the space coordinate;
$\beta$ is a factor;
m is the mass;
c is the specific heat.

### Subscripts

1 denotes the ice;
2 denotes the vapor.

## LITERATURE CITED

1. G. Carslow and D. Jaeger, Heat Conduction in Solids [in Russian], Nauka, Moscow (1964).